



**Recorded Data**

### ***Measurement of Earth's Magnetic Field at Rochester, NY***

(Note: The quoted uncertainties are as optimistic as possible, in order to make the agreement with NOAA data more plausible – see below. The best estimates are undoubtedly larger).

The axis of a square 500 turn coil passing through the mid points of opposite sides was horizontal and oriented east-west. Thus the measured  $|B|$  is the component pointing north. Each side of the coil was  $60 \pm 0.2$  cm long, as measured half-way through the thickness of the windings (about 3 cm). The coil was rotated manually at as constant a rate as possible. The output  $V(t)$  is shown in the figure above.

The average and standard deviation of the 13 maximum voltages between 1.5 and 17 seconds were  $(+0.0530 \pm 0.0033)$  V and those of the 14 minimum voltages were  $(-0.0510 \pm 0.0051)$  V. The average and standard deviation of these extrema were  $(+0.002 \pm 0.006)$  V where  $0.006 = \left[ (0.0033)^2 + (0.0051)^2 \right]^{1/2}$ , so that the average is zero within the uncertainty. The average and standard deviation of all 27 extreme voltage magnitudes are  $(0.0520 \pm 0.0043)$  V. In summary:

Amplitude  $V_{\max} = 0.0520 \pm 0.0043$  V,

Area  $A = (0.360 \pm 0.002)$  m<sup>2</sup>,

Period  $T = 1.16$  s ; angular frequency  $\omega = 2\pi/T = 5.42$  rad/sec (negligible uncertainty),

Number of turns  $N = 500$  (negligible uncertainty).

The computed magnetic field is

$$B_{\text{average}} = V_{\text{max}} / (N \cdot A \cdot \omega) = 0.0520 / [(500)(0.360)(5.42)] = 5.330 \times 10^{-5} \text{ Tesla (T)}$$

and the fractional uncertainty is

$$\begin{aligned} \delta B / B &\approx \left[ (\delta V / V)^2 + (\delta A / A)^2 \right]^{1/2} = \left[ (0.004 / 0.052)^2 + (0.002 / 0.36)^2 \right]^{1/2} \\ &\approx (0.004 / 0.052) = (0.0769) \\ &\Rightarrow \delta B = (0.0769)(5.33 \times 10^{-5} \text{ T}) = 0.41 \times 10^{-6} \text{ T}. \end{aligned}$$

Thus  $|B| = (5.33 \pm 0.41) \times 10^{-5} \text{ T}$ .

The NOAA website states that at Rochester NY  $|B| = 5.378 \times 10^{-5} \text{ T}$  (it is actually given to 6 significant figures) and is directed  $69^\circ 20'$  below horizontal and  $11^\circ 37'$  west of north. Thus the north component is  $(5.378 \times 10^{-5}) \cos(11.5^\circ) = 5.270 \times 10^{-5} \text{ T}$ . The difference between the NOAA and measured north component of the field is about 2%, fortuitously small compared with the estimated 8% uncertainty in the measured value (that as noted above is probably too small). The predicted east-west component is  $(5.378 \times 10^{-5}) \sin(11.5^\circ) = 1.07 \times 10^{-5} \text{ T}$  that corresponds to 0.010 V for the coil considered here. This would have been measurable with the equipment at hand but time ran out.